Natural illumination solution for rotating space settlements

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Abstract

Cylindrical kilometre-scale artificial gravity space settlements were proposed by Gerard O'Neill in the 1970s. The early concept had two oppositely rotating cylinders and moving mirrors to simulate the diurnal cycle. Later, the Kalpana One concept exhibited passively stable rotation and no large moving parts. Here we propose and analyse a specific light transfer solution for Kalpana One type settlements. Our proposed solution is technically reliable because it avoids large moving parts that could be single failure points. The scheme has an array of cylindrical paraboloid concentrators in the outer wall and semi-toroidal reflectors at the equator which distribute the concentrated sunlight onto the living surface. The living cylinder is divided into a number of φ-sections (valleys) that are in different phases of the diurnal and seasonal cycles. To reduce the mass of nitrogen needed, a shallow atmosphere is used which is contained by a pressure-tight transparent roof. The only moving parts needed are local blinders installed below the roof of each valley. We also find that settlements of this class have a natural location at the equator where one can build multi-storey urban blocks. The location is optimal from the mass distribution (rotational stability) point of view. If maximally built, the amount of urban floorspace per person becomes large, up to 25,000 m^2, which is an order of magnitude larger than the food-producing rural biosphere area per person. Large urban floorspace area per person may increase the material standard of living much beyond Earth while increasing the total mass per person relatively little.

Keywords: cylindrical space settlement, O'Neill type space settlement, solar system colonisation

Nomenclature

$A_\perp$	Cross-sectional area
au	Astronomical unit, 149 597 871 km
F_y	Wall tension force
g	Acceleration due to gravity, 9.81 m/s^2
h	Wall thickness
K	Concentration factor
L_z	Settlement length
m	Mass
p	Half slitwidth (*semilatus rectum*) of concentrator
R	Radius of the living wall
x, y, z	Cartesian coordinates
ρ	Radial cylindrical coordinate, $\rho = \sqrt{x^2 + y^2}$
φ	Angular cylindrical coordinate, $\varphi = \mathrm{atan}(y/x)$
ρ_w	Wall mass density (kg/m^3)
σ	Wall tension (Pa)

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1. Introduction

Gerard O'Neill was among the first to propose that people could live on the insides of kilometre-scale spinning cylindrical settlements, located in space and constructed from asteroid or lunar materials [1, 2]. O'Neill's original concept had two cylinders rotating in opposite directions and mechanically connected to each other to make a system with almost zero total angular momentum so that it can be turned to track the Sun. The drawback of such design is the existence of large rotary joints and moving mirrors that are potential sources of single-point failures. Later, the Kalpana One model was proposed [3], which consisted of a single cylinder whose axis of rotation is perpendicular to the orbital plane, thus eliminating much of the mechanical complexity.

It has been pointed out [1, 2, 3] that unlike Moon and Mars, artificial rotating settlements are able to provide an earthlike 1g gravity environment for the inhabitants which ensures that children grow as strong as on Earth so that they are free to visit or move back to Earth as adults if they wish. Also, in the long run, there is enough small body material in the asteroid belt, and even more in the Kuiper belt and beyond, that the total living area of settlements could eventually exceed the surface area of Earth by a large factor. This is understandable because a settlement needs only $\sim 10^4$ kg/m^2 of radiation shielding mass per sunlit living wall area, while an earthlike planet needs a million times more. The settlement technology allows one to build $1g$ living space in the solar system by using the mass per area equivalent to Earth's atmosphere only, rather than the entire planet.

On Moon and Mars it is possible to build radiation shielded and pressurised living space underground e.g. in existing caves or lava tunnels, with low expenditure of processed structural materials such as steel. In contrast, rotating cylindrical settlements in free space need more structural materials and therefore they depend on the existence of scaled-up asteroid mining, space manufacturing and low-cost in-space propulsion such as electric sail [4] for moving the materials and equipment. However, there seems to be no reason why asteroid mining and space manufacturing, once started, could not scale up exponentially.

While O'Neill's groundbreaking idea of rotating cylindrical "inverted planets" received significant public attention and gave rise to conferences and workshops where different settlement geometries were considered (for a review, see Marotta [5]), the peer-reviewed literature on cylindrical rotating settlements is rather small. In addition to the references already mentioned, an example of this literature is a study of the ultimate limit in settlement radius with advanced materials [6].

In this paper we study a class of Kalpana One type settlements [3], but with a specific arrangement for transferring sunlight to mimic earthly diurnal and seasonal cycles without large moving parts. The O'Neill and Kalpana One cylinders were assumed to be filled with breathable atmosphere, but here we assume a relatively shallow (50 m) atmosphere under a transparent pressure-holding roof in order to reduce the amount of nitrogen required. Nitrogen is a relatively rare element on asteroids and on the Moon. Although humans could also survive in a reduced pressure oxygen atmosphere, such atmosphere would introduce an elevated risk of fire. Furthermore, flying insects and birds would have challenges in keeping aloft in a reduced pressure pure oxygen atmosphere, because the mass density of such atmosphere would be much lower than at sea level on Earth. Flying animals play important roles in the internal ecosystem, for example flying insects are responsible for pollination of fruits and berries. In addition to being more affordable in terms of nitrogen, elimination of a cylinder-filling pressurised atmosphere reduces the tensile strength requirement of the walls significantly.

For the location of the settlement, we assume a 1 au orbit, for example the Earth-Moon L4 or L5 Lagrange point or the Earth-Sun L4 or L5 Lagrange point. More general orbits would be possible, but their study is not in the scope of this paper. We assume $1g$ artificial gravity, 1 bar atmospheric pressure and earthlike radiation protection provided by 10^4 kg/m^2 thick walls.

The focus of the paper is a future where – we assume – common engineering materials such as steel, aluminium and carbon fibre composites are available in scaled-up abundance from asteroid or lunar resources, and where the question of the preferred settlement size (the unit size of large-scale solar system settling) is relevant. Most inhabitants likely prefer a large settlement if given the choice. However, the tensile strength requirement of the settlement wall grows linearly with the settlement radius. Thus for a larger settlement, a larger fraction of the wall mass must be structural material instead of biospheric payload like soil, vegetation and people. On the other hand, the radiation shielding requirement sets a minimum areal mass density for the wall which we assume to be 10^4 kg/m^2. Then there exists a settlement size – the sweet spot – where the wall thickness driven by structural requirements also provides the right amount of radiation shielding. Smaller settlements would need equally thick walls because of the radiation shielding requirement and thus they would have the same cost per inhabitant, if cost is measured by the needed asteroid or lunar mass. Larger settlements, on the other hand, would need thicker walls because of the structural requirements, and thus higher mass expenditure per person.

Concerning settlement scale, in this paper we adopt 5 km settlement radius as representing an order of magnitude relevant for long-term future, see Appendix A for wall tensile calculations. There exists the timely question of which size of settlements one should build in near future [7], but such question is outside the scope of the present paper. Indeed, we stress that our choice of 5 km radius should *not* be taken as a recommendation for the first settlement. Our lighting solution is independent of the settlement size, however, and thus it could be applicable also for smaller settlements in near future.

An overarching motivation of the paper is to find a long-term reliable settlement architecture which enables natural sunlight and earthlike (and configurable) diurnal and seasonal illumination cycles. We take the long-term reliability requirement to imply that large moving parts and single failure points must be avoided and that the rotation must be passively stable. As we will find out below, a lighting solution and the associated settlement architecture exists that satisfies the requirements.

The structure of the paper is as follows. We describe the settlement geometry, present a ray-tracing simulation to calculate how sunlight photons are distributed inside the settlement, then discuss the placement of urban blocks[1]. We find that the naturally buildable urban floor area per inhabitant is large, which tends to promote the standard of living. We close the paper by discussion, summary and conclusions. The focus of the paper is on the lighting solution; other features of the settlement are considered as needed. The software used to compute the results is available at [8].

2. Light channel

Directional light such as sunlight can be concentrated e.g. by a parabolic reflector or lens. If the concentrated light is directed into a box (light channel) whose inner walls are perfectly reflecting, the concentrated light fills the entire box because photons can only exit through the same slit where they entered (Figure 1). If the concentration ratio is increased, the light intensity inside the box increases proportionally to it, until reaching the surface brightness of the Sun.

In reality the light intensity is lower because the walls are not ideally reflecting, and the light intensity also depends on the size and shape of the box. For our settlement application, we prefer a light intensity in the light channel which is a few times solar. This is because for safety reasons we want to impose a constraint that if the light channel contains some dark object or dark region (which might be necessary during servicing, for example), the local temperature must not go too high.

Sunlight into the settlement is concentrated by fixed cylindrical paraboloid concentrators into the light channel (Fig. 2). The light channel has a geometric shape that distributes light around the settlement, in particular delivering light also to the antisunward side. The cylindrical rural wall surface (the living cylinder where the biosphere exists) taps its illumination from the adjacent light channel through windows that form the local roof of the rural surface. Local controllable blinders are used in the windows to implement desired diurnal and annual light cycles, in each φ-zone of the rural wall separately. When closed, the blinders reflect light back to the light channel where it remains usable for other φ-zones whose blinders are open.

[1]By urban blocks we refer to multi-storey floorspace that has no natural illumination.

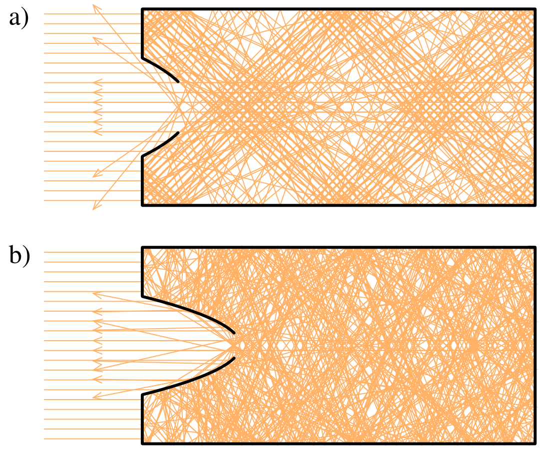

Figure 1: When sunlight is concentrated into a light channel by a parabolic concentrator, photons can only escape through the same slit where they entered. Light intensity inside the box in case (b) is higher than in case (a) because the concentration ratio is higher.

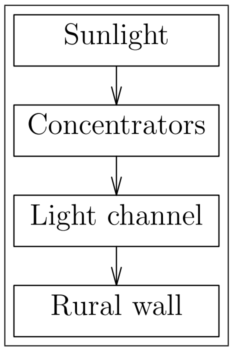

Figure 2: Path of sunlight onto the settlement's rural wall living area.

3. Settlement geometry

A cut 3-D rendering of the settlement is shown in Fig. 3. The overall shape is a cylinder without endcaps which rotates so that the spin axis is orthogonal to the plane of the settlement's heliocentric orbit, i.e. that the spin axis is perpendicular to the Sun direction. (The illumination direction is different in Fig. 3 to aid visualisation of the shape.) The settlement is divided into identical southern and northern cylinders separated by an equatorial region. The cylinder has three walls. The outermost wall has an array of cylindrical paraboloid concentrators, shown as black in Fig. 3, whose purpose is to trap sunlight into the light channel beneath. A concentrator centred at $z = z_1$ is the parametric surface

$$\rho(\varphi, z) = R + \frac{(z - z_1)^2}{2p}, \varphi \in [0, 2\pi), |z - z_1| \in [p, Kp] \quad (1)$$

where p is the half-width of the concentrator slit (the *semilatus rectum* parameter of the parabola) and $K = 20$ is the concentration ratio.

The middle wall is the cylindrical rural wall whose radius is taken to be 5 km. The inner wall reflects the concentrated sunlight that arrives from the equatorial semi-toroidal reflector[2] towards the rural wall.

The biosphere of the settlement resides on the inner surface of the cylindrical rural wall. About 50 m above the living surface there is a pressure-tight transparent roof equipped with 0–100 % adjustable blinders. The nitrogen-oxygen atmosphere has not more than 50 m thickness to limit the mass of nitrogen needed. The biosphere is divided into 20 compartments called valleys in the φ direction. The diurnal and seasonal cycles are simulated by controlling the blinders in each valley separately. The neighbouring valleys have approximately opposite diurnal and seasonal phases so that the total dissipated power stays approximately constant. The yearly average insolation is 100 W/m^2 at the high latitude end of the valley and increases linearly to 160 W/m^2 towards the equatorial end. The values 100 and 160 W/m^2 correspond to yearly average insolation in Helsinki and in southern France, respectively. We assume average albedo of the biosphere of 0.2.

During the simulated night of the valley, the blinders of the valley are closed. Closed blinders reflect photons back into the light channel so that the photons benefit other valleys whose blinders are open. During daytime the blinders are opened partially to simulate the wanted light level.

The equatorial part of the settlement has semi-toroidal reflectors whose purpose is to turn the flux of concentrated sunlight coming from the paraboloid reflectors by approximately 180°. The equatorial region has a belt of solar panels shown as dark blue in Fig. 3 to provide electric power for the settlement.

The interior of the cylinder has two docking ports for external spacecraft. Each docking port is a cylinder with one open end. All parts rotate with the same angular speed so the cylinder walls of the docking ports experience a small artificial gravity of 0.074 g. An arriving spacecraft flies inside the cylinder and sets its landing gear wheels into rotation that matches the rotational velocity of the docking port wall. Then it moves slowly to the rotating wall, perhaps uses some amount of magnetic attachment to prevent bouncing, and applies braking to the wheels. Due to the braking, the spacecraft gradually starts to co-move with the wall and the centrifugal force increases, ensuring that the wheels stay firmly on the wall even without magnetic attachment. Finally the spacecraft comes to rest with respect to the rotating wall and experiences the same centrifugal acceleration (artificial gravity) than the wall. The system can be thought of as a cylindrical landing strip where wheeled craft of different sizes may land.

The edge of the cylinder contains one or more inclined ramps. A departing spacecraft rolls over the edge of the landing port cylinder along a ramp, like a wheeled aeroplane that starts to accelerate downhill. After exiting the ramp and unless propulsively changed, the spacecraft follows a straight path and moves with the rotational velocity of the docking port wall. The docking port is placed outside the z range of other structures of the settlement so that the departing spacecraft does not collide any obstacle.

The arrival and departure procedures sketched in the previous two paragraphs are such that the spacecraft need only low-thrust proopulsion. Impulsive chemical propulsion is not mandatory, which is good from the safety point of view. If chemical propulsion is nevertheless used for some external reason and if an accidental explosion occurs in the docking port, the open cylinder shape of the docking port tends to direct the explosive energy away from the settlement. Existence of two docking ports enables access to the settlement through the other port while a damaged port is repaired.

The docking ports are connected with a central hub by train tubes that carry passengers and cargo. The central hub contains a recreational low-g space. It is connected with the cylindrical artificial gravity surface by elevator tubes.

Thick walls that play the role of a radiation shield are shown in reddish colour in Fig. 3.

A 2-D cross section of the settlement is shown in Fig. 4, where the radiation shielding parts are marked red. The radiation shielding walls are positioned so that no direct paths exist between the living wall surface and external space.

The main parameters of the settlement are listed in Table 1. The light transfer strategy is scalable to any settlement size. The main underlying assumptions are summarised in Table 2.

We adopt a population density of 500 persons per square kilometre of rural wall area. For comparison, The Netherlands has a population density of 400/km^2 and is a net exporter of food. It is a requirement that a space settlement produces all the food that it consumes in a closed ecosystem, with sufficient margin.

The settlement length is selected by requiring, fol-

[2] A cylindrical corner reflector made of two conical surfaces was also tried, but seemed to distribute light to the backside somewhat less efficiently than the toroidal version.

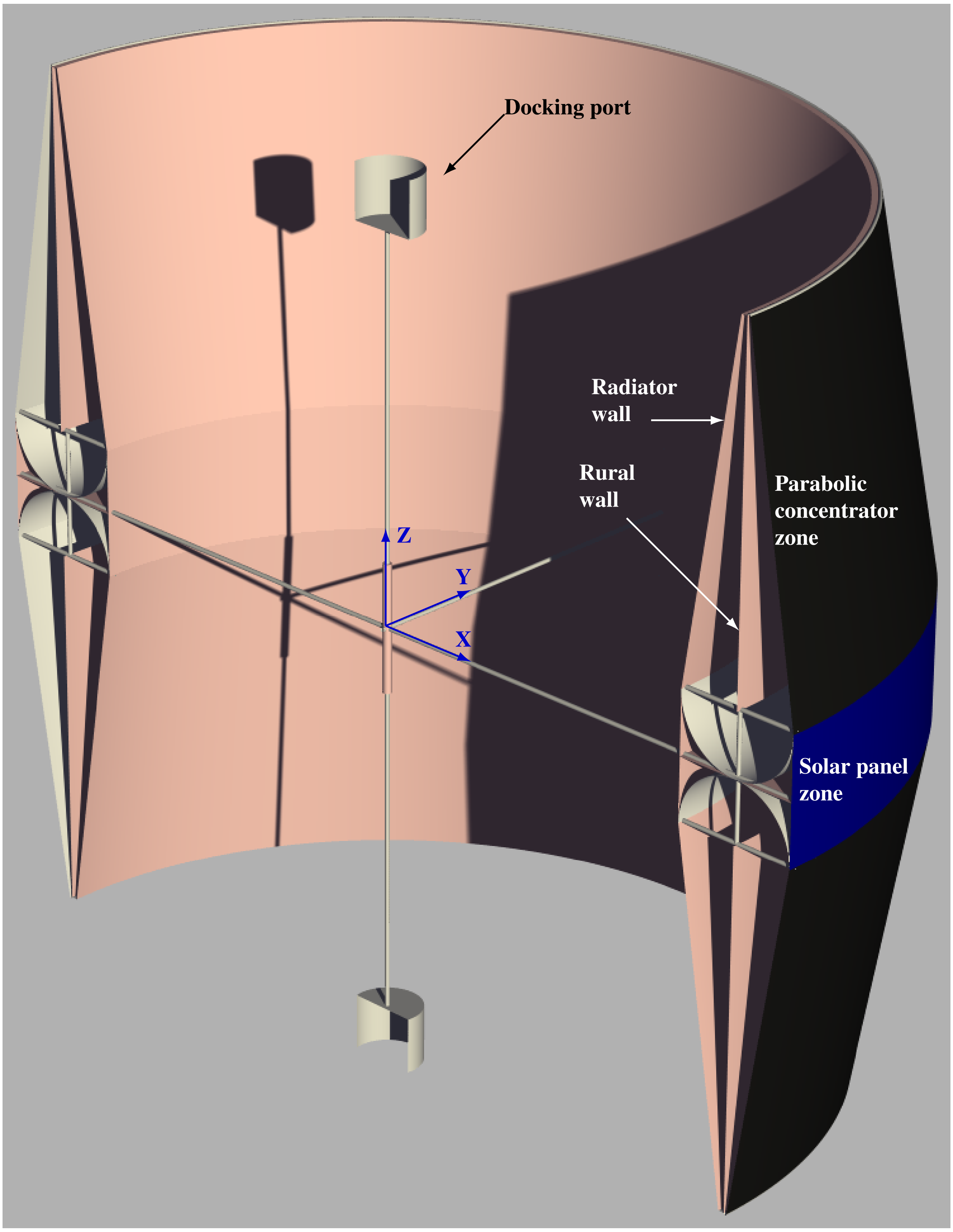

Figure 3: Cut view of the cylindrical space settlement.

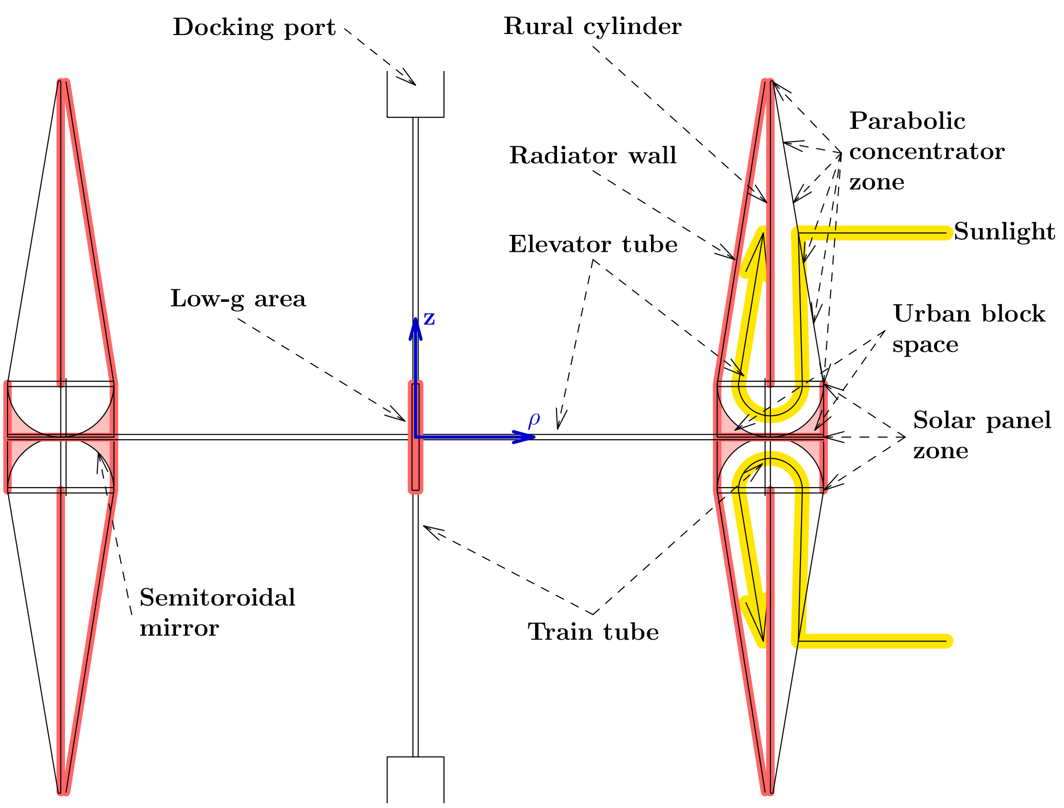

Figure 4: 2-D cut view of the cylindrical space settlement. Thick radiation shielding parts of the walls are shown in red.

Table 1: Main parameters of the settlement.

Diameter	11.5 km
Length	10.01 km*
Radiation shield wall mass	$3.55 \cdot 10^{12}$ kg
Total mass	$\sim 4 \cdot 10^{12}$ kg
Rural wall radius	5.0 km
Rural wall length	2×4.255 km
Rural area	267 km^2
Number of valleys	2×20
Valley width	1.57 km
Valley length	4.255 km
Valley area	6.68 km^2
Reflector semitorus radius	750 m
Sun-facing solar panel area	17.25 km^2 (11.5 km $\times$ 1.5 km)
Electric power /w 20 % solar panel efficiency	4.7 GW
Artificial gravity at rural wall	0.93 g
Rotation period	2.45 min

* Excluding docking ports.

Table 2: Underlying assumptions.

Solar distance	1 au
Solar constant after filtering out IR,UV	1000 W/m^2
Inertia moment ratio I_{zz}/I_{xx}	1.2
Mirror absorptivity	0.02
Mirror diffuse reflectance fraction	0.001
Concentration factor, K	20
Maximum rural time-average insolation	160 W/m^2
Minimum rural time-average insolation	100 W/m^2
Spatiotemporal average rural insolation	130 W/m^2
Rural albedo	0.2
Radiation shielding mass*	10^4 kg/m^2

* Doubles as structural mass.

lowing the Kalpana One design choice, that the inertial moment tensor component I_{zz} is 20 % larger than the x and y components: $I_{zz} = 1.2I_{xx}$. This is thought to be enough to have a passively stable rotation with sufficient margin. Here $I_{xx} = I_{yy}$ holds because of the cylindrical symmetry. Only the radiation shielding walls were considered when calculating the inertial moment, because they are the heaviest parts.

Figure 5 shows 2-D cross sections of the settlement with the paraboloid concentrators in different scales, i.e. different *semilatus rectum* parameters p. In an actual settlement one could have a large number of small concentrators (small p, perhaps a few centimetres). Simulating a small p is time-consuming and in the calculations we use $p = 1$ m (Fig. 5c). We have verified that the results are not sensitive to the value of p.

4. Illumination simulation

We wrote a `C++14` [9] code that performs forward and backward ray-tracing. Figure 3 was produced by the code by backward ray-tracing. Forward ray-tracing is used to launch $4 \cdot 10^6$ random solar photons towards the settlement. Each photon typically undergoes many specular and diffuse reflections until it is absorbed by some of the optical surfaces or by the settlement's biosphere. The photon may also exit back to space through a paraboloid concentrator slit. The ray-tracing code supports geometric shapes such as planes, cylinders, cones, tori and cylindrical paraboloids, as well as the set operations union, intersection, complement and difference to form more complicated surfaces.

We start with a trial value of 0.08 for the biosphere's absorptivity. The absorptivity value models both the blinders and the biosphere itself. The resulting absorbed power density is calculated as a gridded function of φ and z on the cylindrical rural wall. We then calculate the corresponding insolation (power density that would be absorbed by a small piece of fully absorbing material) by dividing the actually absorbed power density by one minus the albedo of the biosphere, for which we use the value 0.2. We compare the obtained insolation with the wanted insolation which varies between 100 and 160 W/m^2 in z. Then we multiply the local absorptivity by the ratio of the wanted versus obtained insolation and compute the next iteration. We perform four iterations, during which the result typically converges well. As a result, the code reproduces the wanted insolation profile, using some absorptivity profile that depends on φ and z.

We show the results in Fig. 6. Figure 6a shows the incident illumination above the blinders, which is calculated by dividing the absorbed power density by the local absorptivity. (We checked that we obtain the same result from a transparent photon-counting detector near the surface.) The incident illumination represents the highest obtainable illumination that results if all blinders are fully open and the roof is completely transparent. For most φ values the incident illumination exceeds 1000 W/m^2. The value 1000 W/m^2 corresponds to sunshine on a cloudless day when the sun is at the zenith. The lowest illumination that occurs for $\varphi \approx 180°$ is 850 W/m^2. Each point on the biosphere passes through all φ values in a few minutes as the settlement rotates. Hence, for each z the minimum incident flux corresponds to the maximum obtainable illumination, if one wants to avoid periodic dimming of the daylight at the rotational frequency.

Figure 6b is the corresponding time-averaged absorption, which includes the effect of the blinders as well as the biosphere itself. The lowest panel Fig. 6c is the obtained insolation below the blinders. Apart from numerical noise originating from the finite number of photons used in the calculation, the result corresponds to the wanted insolation profiles that goes from 160 to 100 W/m^2 when one moves from the equatorial end to the high latitude end.

The cylindrically symmetric light channel distributes light in φ and z. Figure 7 shows paths of photons that enter the settlement in a particular exemplary y = const plane ($y = 0.5R$). To ease visualisation, views from two directions are included. Some photons are reflected out already when interacting with the cylindrical paraboloid concentrator. A few photons find their way out from the concentrator slits later after reflecting back and forth inside the light channel. A significant amount of photons go to the antisunward side, some even make a full circle around the settlement. The Monte Carlo approach was

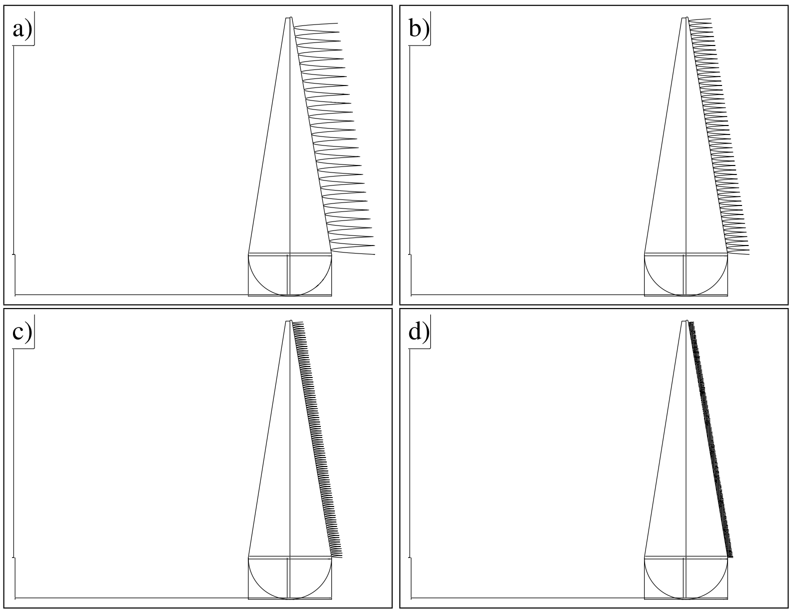

Figure 5: 2-D cuts showing the parabolic concentrators for different slit half-width parameters p of 4 m (a), 2 m (b), 1 m (c), 0.5 m (d). The value $p = 1$ m (panel c) is used in the calculations. In the limit of small p (small and numerous concentrators), the parabolic concentrator zone looks dark and featureless to an external viewer, as was depicted in Fig. 3.

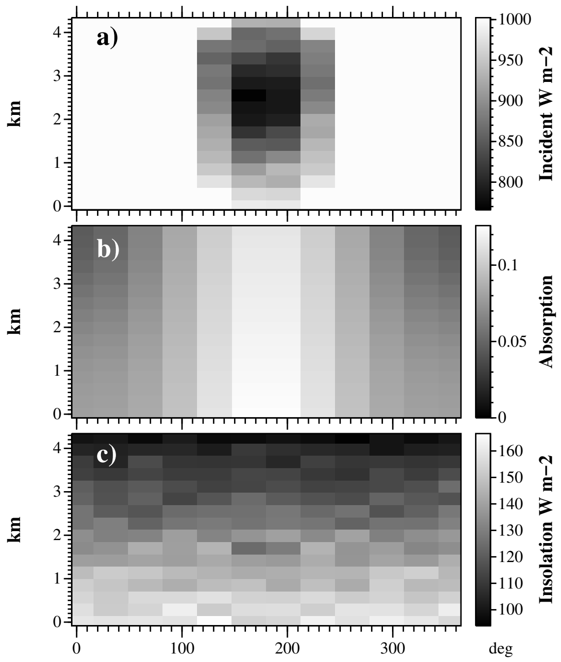

Figure 6: Incident photon power density above the blinders (a), effective absorptivity of the blinder-equipped settlement surface (b), insolation below the blinders. All are as function of the cylindrical coordinates φ and z. In (a), values over 1 kW are shown are white.

used in the simulation, i.e., at each reflection the photon also has a chance of being absorbed at a nonzero probability. In Fig. 7, the path of each photon was traced until it escaped or was absorbed at an internal wall. The internal walls include reflector surfaces that are part of the light channel, blinders covering the windows between the light channel and the rural wall, and the rural wall itself.

The cylindrical paraboloid concentrators are somewhat sensitive to the direction of the incoming sunlight. Increasing the concentration ratio K (i.e. the total width versus slit width of the parabola) traps light better because it reduces the amount of light that escapes through the slits. On the other hand, increasing K increases the directional sensitivity. We use $K = 20$ as a tradeoff. Figure 8 shows the total sunlight power absorbed by the rural biosphere (in the $z > 0$ half of the settlement) as a function of the angle between the sun direction and the spin plane. We see in Fig. 8 that if the spin axis is controlled with better than $\sim 0.5°$ precision, then the absorbed power is optimal, while a deviation of 2 degrees causes up to 20 % reduction. Even a 20 % reduction is not catastrophic for the crop yields, however, since plants are accustomed to cloudiness associated insolation variations on Earth during the growing season. The temperature remains controllable by regulating the cooling through the blinders and the radiator wall. We think that the sensitivity to spin axis inclination is not

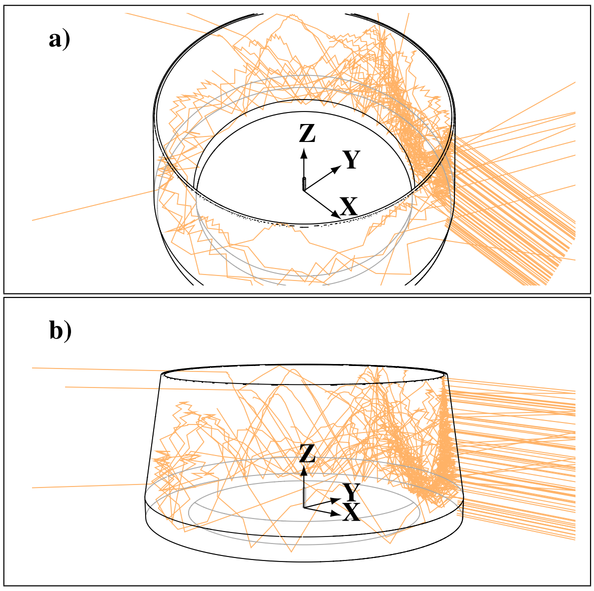

Figure 7: Photon paths inside the light channel from two viewing angles. A vertical sheet of input photons enters from the right hand side and propagates along $-\hat{x}$ at $y = R/2$, while z is a random number in interval $[0, Lz/2)$.

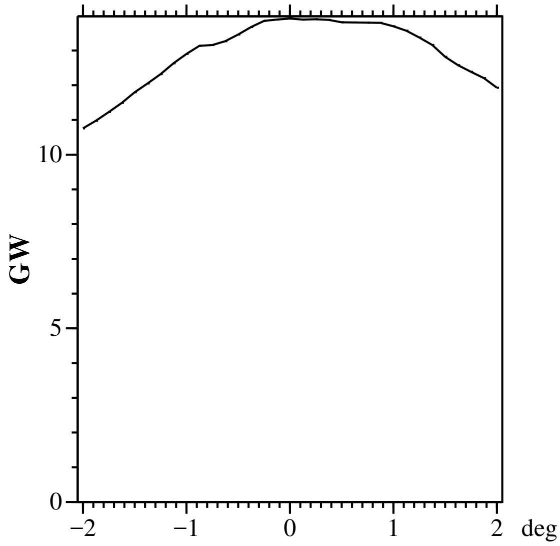

Figure 8: Power absorbed by the living cylinder as function of sun angle.

a problem. The sun angle dependence (Fig. 8) is not quite symmetrical because although the southern and northern parts of the settlement are identical, there is no light exchange between them and the northern part taken alone is not symmetrical in z.

The settlement is cooled radiatively through the inner conical reflector wall whose interior emits thermal infrared into space. A glass-covered optical mirror of the inner side of the conical surface absorbs well in thermal infrared and conducts heat away because its underlying support structure is metallic. To avoid using heat pumps, the radiator temperature must be lower than the dewpoint of the climate that one wants to maintain. In our simulation, the average radiator temperature is −16 °C if one makes the approximation that the effective radiating areas are the open endcap areas of the cylinder (making such assumption models the effect that part of the emitted infrared is re-absorbed by the opposing radiator wall). Power generated at the rural wall is radiated away by the nearby radiator wall so that liquid transfer of heat over longer distances is not required.

5. Urban blocks

The population density per rural wall area is limited by the biological productivity of the rural walls, we have assumed 500 persons per square kilometre, or 2000 m^2 of rural area per person. Additional space per person can be obtained by building urban type multistorey blocks, and it can be done without reducing the rural area. For example, urban blocks could simply exist on the rural walls so that the roofs of the buildings are used for agriculture. Another possibility which is better from the mass distribution point of view is to build urban blocks inside the volume (Fig. 4, pale red areas) that is left over by the equatorial semi-toroidal reflectors. Equatorial mass promotes rotational stability, and the extra stability would enable making the settlement somewhat longer, with a corresponding increase in rural area and consequently the maximum population.

Table 3 lists population related parameters. For a settlement of 5 km rural wall radius, the space available for urban blocks between the toroidal reflectors (pale red areas in Fig. 4) is as large as 15 km^3. The volume scales cubically with the settlement radius. The volume corresponds to a vast $25,000$ m^2 of urban floor area per person, which is an order of magnitude larger than the rural area per person. In this paper we call this the maximal (or naturally buildable maximal) urban area, even though it is not really maximum because one could build also additional urban spaces in the rural areas.

To have a rough estimate of the mass of urban blocks, the large luxury cruise ship "Oasis of the Seas" weighs 10^8 kg and its total internal volume is $710.6 \cdot 10^3$ m^3, as calculated from the gross tonnage of 225,282 GT[10], yielding a bulk density of 141 kg/m^3. If this bulk density is representative of the settlement blocks as well, the mass of the fully built urban blocks filling 15 km^3 volume is $2 \cdot 10^{12}$ kg, which is 50 % of the mass of the rural-only version of the settlement (Table 1). If one chooses to build 10 % of the maximum urban volume, for example, it increases the settlement mass by only 5 % and provides 2000 m^2 of urban floor area per person, which is still a vast amount.

Table 3: Population parameters.

Total population	134,000
Population density per rural area	500 km^{-2}
Rural area per capita	2000 m^2
Electric power per capita	35 kW
Max equatorial urban block volume	15 km^3
Max urban floor area *	$3.3 \cdot 10^9$ m^2
Max urban volume per capita	$110,000$ m^3
Max urban floor area per capita	$25,000$ m^2
El. power per fully built urban area	1.5 W/m^2
Artificial gravity in urban blocks	0.79–1.07 g

* Assuming 4.5 m floor-to-floor height difference.

The available electric power is large per capita (35 kW), although lowish per maximal urban floor area (1.5 W). The artificial gravity in the inner and and outer urban volume is lower and higher, respectively, than on the rural walls. In Tables 1 and 3 we have selected the $1g$ level to be reached halfway between the rural wall and the outermost part of the urban blocks. For example, kindergartens might exist in the $\geq 1g$ parts so that children grow as strong as on Earth, whereas motion-challenged old people might live in the parts were the gravity is lower than $1g$.

The interiors of the urban blocks should also be shielded against radiation. We do not include mass for such shielding in the mass budget explicitly, because we consider it likely that parts of the urban blocks are used as storage area for items that do not need radiation protection. By placing the storage areas at the outer boundary of the urban block, their mass contributes to the shielding of the inside of the block so that dedicated radiation shielding might be unnecessary.

6. Discussion

We assumed a 5 km settlement radius. The value is meant to correspond roughly to the sweet spot where the structural walls double as radiation shields. Settlements of all sizes up to and including the sweet spot size have the same wall thickness and thus the same mass cost per inhabitant[3]. The sweet spot size is preferred because a large settlement is preferable over a small one. Settlements larger than the sweet spot size have, however, a higher mass cost per inhabitant because of structural requirements. Because of this, we consider it likely that large-scale settling of the solar system is undertaken using settlements around the sweet spot size.

We assumed light channel width which is 15 % of the rural wall radius (750 m, since the rural wall radius is 5 km). A wider light channel would distribute sunlight more evenly, but would increase the mass needed per illuminated rural area because a larger fraction of the settlement length would be radiation shielding surface.

Periodic variation of the insolation by the rotation rate is not observable by the inhabitants because it is suppressed by continuous active controlling of the blinders. The diurnal and seasonal cycles are produced synthetically by the blinders. Failure of a small fraction of the blinders is acceptable. The blinders can be lubricated in a normal manner because they reside in a 1 bar pressure normal atmosphere in normal temperature. They can be serviced and replaced by humans and robots easily because they reside in earthlike gravity and radiation protection.

We divided the cylindrical rural wall into (for example) 20 valleys in the φ direction. Each valley is in its own phase of the diurnal and seasonal cycle. The arrangement enables simulating the diurnal and seasonal cycles using local blinders while keeping the total sunlight power dissipation roughly constant. Neighbouring valleys are isolated from each other optically, and for safety reasons they can also be pressurised independently of each other. Division of the rural wall cylinder into valleys also has the benefit of making the cylinder's curvature visually less apparent to the inhabitants, particularly if there are artificial ridges or "mountains" between the valleys.

One benefit of the proposed architecture is that thick radiation and meteoroid protecting transparent windows are not needed.

It is noteworthy that the naturally available maximal urban floor area per inhabitant is large, 25,000 m^2 under the baseline assumptions. This floor area per person is 2-3 orders of magnitude larger than the contemporary average on Earth and comparable to what today's royal families have access to. If the urban blocks are maximally built, electric power usage per urban floor area must be limited to 1.5 W/m^2. However, this does not

[3]The sweet spot size is a function of the radiation environment. Thus an equatorial LEO settlement would have a much smaller sweet spot size than the deep space settlement considered in this paper.

restrict the use of the floorspace in an essential way because rooms where no people are present do not need energy. Plants feeding the biosphere must be grown in the open rural areas where the light energy dissipation per area is two orders of magnitude higher. The dominant power is the sunlight dissipated in the rural areas, while for a maximally built settlement the dominant floor area exists in electrically lighted urban blocks.

The $3.3 \cdot 10^9$ m^2 urban floor area is so large that if arranged in rooms of 6.7 m wide, for example, a path going through all the rooms must be at least $5 \cdot 10^5$ km long. A person walking 20 km per day for 70 years could theoretically visit every room once. Thus, a 5 km radius settlement can have more things inside than an inhabitant can experience or explore over an entire lifetime. Even so, the maximal urban volume is only $\sim 6\,\%$ of the volume of the light channels.

We calculated the length of the settlement from the requirement $I_{zz} = 1.2 \cdot I_{xx}$, without including the mass of any urban blocks. If one decides to introduce large urban blocks, the settlement could be made somewhat longer while keeping passively stable rotation. It is also possible and perhaps likely that urban blocks are constructed gradually by the first generations of inhabitants. In that case, the settlement length must be restricted so that rotation is stable also without the mass of the urban blocks, which is the case that was calculated in this paper.

The inner and outer light channels are tapered, i.e. the channel gets narrower with increasing $|z|$ as seen e.g. in Fig. 4. A tapered inner light channel works better than a non-tapered one, for multiple reasons:

1. According to numerical experimentation, a tapered inner light channel distributes light better onto the rural wall. The sunlight power that reaches the rural wall is higher and more uniform in φ and z.
2. A non-tapered inner light channel would need to be closed by a radiation shielding end member. The radiation shield would increase the settlement's total mass, and the extra mass would be located at high $|z|$ which would reduce the stability of the rotation. To restore stable rotation, the cylindrical rural wall would have to be made shorter, which would result in smaller area for the food-producing biosphere and thus a smaller maximum population.
3. Cooling of the settlement occurs by thermal radiation emitted by the inner wall of the inner light channel[4]. The emitted thermal radiation is partially re-absorbed by the opposing wall. To account for the re-absorption, the effective radiator area is approximately given by the area of the open end of the cylinder. The open area is larger if the inner light channel is tapered. Hence, tapering increases the available cooling power of the settlement, which increases the maximum tolerable sunlight power that the rural wall can be configured to absorb by the artificial diurnal and seasonal illumination cycles. Consequently, when all else is equal, the maximum biological production and the maximum allowed human population are larger when the inner light channel is tapered.

Tapering is beneficial also for the outer light channel because according to numerical experimentation, a tapered channel directs light towards the equatorial semitoroidal reflector with fewer reflections, so that less light gets absorbed by the reflector surfaces or escapes through the concentrator slits back into space.

Are there other ways to design a settlement that produces earthlike illumination cycles without large-scale moving parts? One approach would be to use artificial light sources instead of natural sunlight and to produce the needed electric power by covering the exterior surface by solar panels. However, natural sunlight is expected to be visually more pleasing than artificial light and it also has cost, lifetime, reliability and thermal management benefits relative to electric lighting.

7. Summary and conclusions

We have presented a cylindrical space settlement concept where sunlight is concentrated by cylindrical paraboloid concentrators and reflected by semi-toroidal and conical reflectors and controlled by local blinders to simulate earthlike diurnal and seasonal illumination cycles. The rural wall living cylinder is divided into 20 (for example) z-directed valleys which are in different phases of the light cycles. No moving parts are needed other than numerous and easily accessible local blinders that regulate light input into the valleys. The settlement rotates as a rigid body and the mass distribution is such that the rotation is passively stable.

The geometry has natural spare volume at the equator where one can add multi-storey urban blocks without reducing the rural area. Adjacent to the urban blocks there is natural place where to install a zone of solar panels without reducing the amount of gathered light.

The inhabitant population is limited by the ability of the closed ecosystem of the sunlit rural areas to produce food. The naturally buildable total urban floorspace area

[4] In this cooling method, heat produced at the rural wall by dissipated sunlight is radiated away by the adjacent inner lightchannel surface so that there is not much if any need for fluid-based heat transfer.

can be an order of magnitude larger than the rural area if the settlement radius is 5 km. The maximum urban to rural area ratio scales linearly with the settlement radius. A 5 km settlement radius corresponds roughly to the sweet design spot where earthlike radiation shielding is produced for free by the required structural mass.

Overall, the settlement concepts satisfies the following generic requirements for long-term large-scale settling of the solar system:

1. $1g$ artificial gravity, earthlike atmosphere, earthlike radiation protection.
2. Large enough size so that internals of the settlement exceed a person's lifetime-integrated capacity to explore.
3. Standard of living reminiscent to contemporary royal families on Earth, quantified by up to 25,000 m^2 of urban living area and 2000 m^2 of rural area per inhabitant.
4. Access to other settlements and Earth by spacecraft docking ports, using safe arrival and departure procedures that do not require impulsive chemical propulsion.

In particular, the proposed lighting geometry enables a long-term reliable architecture, i.e., a design that does not include large moving parts, is free of single failure points and exhibits passively stable rotation.

As a future refinement of the concept, one could consider more general orbits than 1 au circular, and a range of settlement sizes could be analysed.

In conclusion, a requirement for settling the solar system in a large scale is that the habitats must be long-term reliable and they should provide a high standard of living compared to Earth. In the light of our analysis, the goals seem possible to reach, without essentially increasing the total mass consumption per inhabitant beyond what is required by radiation protection in any case.

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Appendix A. Wall tension

Consider a cylindrical settlement wall of radius R, length L_z, thickness h and wall mass density ρ_w, rotating so that the centrifugal acceleration at the wall is g. The mass of the cylinder is

$$m = 2\pi R L_z h \rho_w. \tag{A.1}$$

Use cylindrical coordinates ρ, φ and z so that the cylinder is centred at origin and its axis is aligned with $\hat{z}$. Let us consider the $y > 0$ half of the cylinder. For this semicylinder $0 \le \varphi < \pi$.

The wall tension F_y at $y = 0$ is equal to the y-component of the centrifugal force acting on the $\{y > 0\}$ semicylinder:

$$F_y = \int dF_y = \int dmg\sin\varphi = \frac{mg}{2\pi}\int_0^\pi d\varphi \sin\varphi = \frac{mg}{\pi}. \tag{A.2}$$

The force F_y pulls a cross-sectional wall area

$$A_\perp = 2L_z h \tag{A.3}$$

where factor 2 comes from two edges of the semicylinder at $x = \pm R$. The tensile stress σ of the wall is then given by

$$\sigma = \frac{F_y}{A_\perp} = R\rho_w g. \tag{A.4}$$

If $R = 5$ km, $\rho_w = 7.8 \cdot 10^3$ kg/m^3 (steel) and $g = 9.81$ m/s^2, we obtain tensile stress $\sigma = 380$ MPa. In the form of piano wire, steel's ultimate tensile strength can exceed 2500 MPa [11]. If half of the wall mass is payload (soil, vegetation, etc.), for example, then the structural parts must have $2 \times 380 = 760$ MPa of usable tensile strength. This is 30 % of the piano steel wire ultimate strength of 2500 MPa. In the light of these numbers, a living wall radius of 5 km seems feasible. Steel is a reasonable choice for the main structural material since iron is one of the dominant elements on asteroids.

It is of interest to notice that the wall stress (A.4) does not depend on the wall thickness h. This is because we did not include the internal atmospheric pressure of the settlement in the calculation. Neglecting the internal pressure is a good approximation since the 50 m high atmosphere contributes only 65 kg/m^2 to the wall mass loading.

When the settlement radius R is increased, the wall tensile strength requirement σ increases proportionally, according to Eq. (A.4). If the structural material characteristics remain the same, this means that for a larger settlement, a smaller fraction of the wall mass can be payload (soil etc.). If the payload mass per area is kept unchanged, this implies that the wall's total mass per area must increase. The question of what is the sweet spot settlement size (i.e., size where radiation shielding and structural requirements meet) depends on the required payload mass per living wall area. In conclusion, the adopted 5 km settlement radius is compatible with, for example, 50 % wall payload fraction and tensile loading which is 30 % of piano wire steel's ultimate tensile strength.

References

[1] O'Neill, G.K., The colonization of space, Physics Today 27 (9) (1974) 32–40.
[2] O'Neill, G.K., The high frontier, New York (1977) 288.
[3] Globus, A., N. Arora, A. Bajoria and J. Strour, The Kalpana One orbital space settlement revised, American Institute of Aeronautics and Astronautics (2007) http://space.nss.org/media/Kalpana-One-2007.pdf.
[4] Janhunen, P., et al., Electric solar wind sail: towards test missions, Rev.Sci.Instrum. 81 (2010) 111301.
[5] Marotta, T., A survey of space settlement designs, NSS Space Settlement J. (2016) http://space.nss.org/media/NSS-JOURNAL-A-Survey-of-Space-Settlement-Designs.pdf.
[6] McKendree, T.L., Implications of molecular nanotechnology technical performance parameters on previously defined space system architectures, Nanotech. 7 (3) (1996) 204–209.
[7] Globus, A., S. Covey and D. Faber, Space settlement: an easier way, NSS Space Settlement J. (2017) http://space.nss.org/media/NSS-JOURNAL-Space-Settlement-An-Easier-Way.pdf.
[8] Janhunen, P., Raytracing software for this paper (2018) http://www.electric-sailing.fi/papers/paper43/raytrace.tar.gz.
[9] The C++14 language, https://en.wikipedia.org/wiki/C%2B%2B14.
[10] https://en.wikipedia.org/wiki/MS_Oasis_of_the_Seas, accessed on August 25, 2018.
[11] Piano wire of J-WITEX Corporation, http://www.j-witex.co.jp/en/products/wire2/piano.html, accessed on August 25, 2018.